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Traffic flow quality as part of network quality for a sparse road network

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Abstract

Sparse Road Networks are dominant in Indonesia, e.g. the regency, the provincial and the national road network. For these cases, measuring the network quality, apart from the physical qualities of the road segments and the intersections, is very important. The quality measure must be based on how the network can perform its function. Three main road network function as a measure of network quality are : to connect different points, to flow the traffic and to cover the area in a certain density. This paper is designated to present the research result on formulating the traffic flow quality as part of network quality and on developing its calculation method. It can be concluded that the research objectives have been attained, the traffic flow quality has been formulated and its calculation method has been also developed. Two type of measure of traffic flow quality: the flow itinerary quality and the flow fluidity quality. The fluidity quality deals with traffic engineering, which is already a lot developed. On the other hand the flow itinerary quality still has to be formulated. The flow itinerary quality is about whether for doing trip from point A to point B, the existing itinerary is say direct or going around. Thus the quality is formulated by a ratio between the existing itinerary length to the direct distance and a ratio between the existing vehicle-km flow to the direct distance vehicle-km. It needs a special calculation method to be practical. A special matrix calculation has been developed, based on a special matrix form. Bangkalan Regency Road Network has been taken as a trial case.

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1. Introduction

A region must have a good road network. In Indonesia, the road network quality is still normally measure in terms of road segments and intersections quality. This is to measure its physical quality, mainly geometric quality, pavement quality and drainage quality. These are, afterward, related to traffic fluidity, safety, comfort, etc [1,2,3]. But, realized or unrealized, road network physical quality itself is not enough, 'network quality' must be considered, since it is very important [4,5,6]. It can be clearly shown by, e.g. the Jakarta By-pass – to realize a good connection between Tanjung Priok Port to its hinterland, the Pasupati Bridge in Bandung Indonesia – to improve connection between eastern part to western part of Bandung city, the Normandy Bridge in France – to shorten connection between Le Havre to the touristics and industrial area in the south of Seine River. These all were constructed not because the road physical condition was bad, but were built to improve the connectivity, thus to improve the 'network quality' of the road network.

A researches to develop a 'network quality' for a sparse road network has been conducted. This paper is designated to present the result, which can be derived into three main objective : to define network quality measure, to define the network's model and to developed a calculation method. This paper presents only the traffic flow quality part.

2. Literature Review

As the research has three main objectives, the literature review was done mainly on these three items : the transportation network quality, the road transportation model and the related mathematics.

Three different formulation of network quality measure has been accessed : the geography of transport, the location theory and the industrial engineering formulation. Certain geography of transport group in USA introduce the road network quality. The formulation is fully based on graph theory for a planar graph. The network quality consists of several items : manimum connectivity, maximum connectivity, gamma index, network form type, network diameter, degree of node, number of connections for a pair of nodes, shortet path in number of links, shortest path in length [6]. The location theory introduce the accessibility concept of a node in an area [6,7,8]. Certain industrial engineering group in Indonesia introduce network coverage and network density as a measure of road network quality [9]. It can be concluded that there are varieties of network quality measure, but there is no one sound integrated network quality measure. Using graph theory as it is for formulating network quality can lead us to inappropriate formulation.

Road transportation network has been modeled in several ways according to the purpose. The major models are : simple node-link network model, 4 steps modelling network model, traffic flow network model – can be said as TRANSYT model, and cell transmission model [8,10,11,12,13]. The simple node-link network model seems appropriate.

Since the research deals with network problem, the related mathematics to be reviewed are graph theory, network theory, operation research, linear algebra, discrete mathematics, matrix theory and max-plus algebra. Graph theory and network theory consider basically only about the edges connected each other at the vertex. Vertex data, coordinate for example is not considered. The edge data can be presented in a form of a list, in an incidence matrix or in an adjacency matrix [14]. In operation research, several network optimization have been developed. One of these, the shortest path algorithym is developed based on list type data [15,16,17]. In linear algebra, the most used item used in Civil Engineering, the simultaneous linear equations knows three calcaultion methods : elimination, iteration and matrix calculation [18,19,20]. Discrete mathematics is designated for informatics, it deals with all type of integer cases. The shortest path calculation is one of discussions. The same as in operation research, it is solved in heuristic type calculation based on list of data and not based on matrix of data [21]. The matrix theory deals with certain matrix forms and certain matrix operations. These are not enough for the purpose of network analyse [22]. Certain matrix calculation method has been introduced in geography of transport, but the matrix calculation is still not practical [6]. Max-plus algebra, developed in system control domain, is destinated to cope with discrete event system. This deals with special mathematics operation for constant and matrix. The matrix power operation has been shown that it can be used to indicate the existence of a tree network and to plan schedule coordination of a public transport network. Later on, Min-Plus Agebra has been developed based on Max-Plus Algebra [23,24,25,26,27,28].

It can be noted that these mathematical techniques can not answer the overall necessity of special matrix form and special matrix calculation for transportation network analysis.

The above literature review lead us to a following summary notes. A sound integrated network quality still need to be developed. The simple node-link network model seems to be appropriate for this research. A special matrix for network data representation and its calculation method is still need to be developed.

3. Network Quality Concept Development

3.1. Network Quality

The Network Quality must be measured in terms of its performance to realize its function. Therefore the quality derivation must follow this procedure : function identification, performance components (quality item) identification for each function and quality measure development. This is presented in Tabel 1 below.

Table 1. Network Quality Components

Function	Quality item	Explanation
To connect different nodes of the region	connectivity	How well the nodes are connected each other.
	accessibility	How well a node can be accessed from the others and vice versa.
To flow the traffic	traffic flow itinerary	The itinerary quality for each pair of origin-destination
	traffic flow fluidity	The traffic flow performance : speed, delay, stops – traffic engineering
To cover the region	coverage	How well the region is covered
	density	How good is the network density

Apart from that concept, in order to realize the above three main functions, the road network has to have certain supporting qualities as mentioned below :

- Network structure : network category, road functions, road class and road status.
- Physical condition : geometric, pavement, drainage.

To flow the traffic function consists of two quality items : the traffic fluidity and the traffic flow itinerary. The first item is a traffic engineering matter. In Indonesia, it is already a lot developed, even if it still needs certain knowledge development. This quality item is not included in the research object. Contrary, the traffic flow itinerary quality is not yet well defined, even if it is very important. This paper is focusing on this matter.

3.2. Network Model

Among several existing common models for road transportation analysis, the simple node-link network model considered the most appropriate, since detailed traffic flow fluidity performance is not considered on this research.

3.3. Traffic Flow Itinerary Quality

Traffic Flow Itinerary quality deals with whether the existing itinerary to flow from node A to node B can be considered as a direct distance or going around distance. The measure can be defined as follows : the ratio of existing network shorstest path to the direct distance, this can be measured either in distance ratio, in travel time ratio or in vehicle-km ratio. A minimum ratio value for good itinerary than has to be defined. Let's < 2 is taken as a good ratio value.

An calculation result example will be presented for an Example Case presented in Figure 1 below. While the calculation result is presented in Table 2 below.

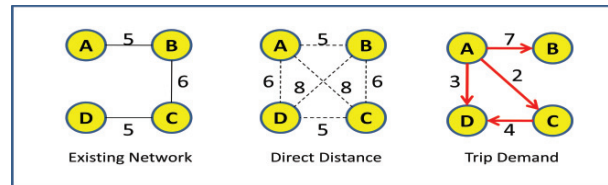


Fig. 1 Example of Traffic Flow Itinerary Quality Case

Table 2. Calculation Result for Example Case – Traffic Flow Itinerary Quality : A-D

Traffic Flow Itinerary Quality : A-D	Itinerary Value		Ratio	Evaluation
	Existing NW	Direct Distance		
Number of Trip	3			
In distance	16	6	2.67	bad
In vehicle-km	48	18	2.67	bad

3.4. Calculation Method

Normally, the observed road network is complex and large. So even if quality formula is simple, it needs a special calculation method, since a manual calculation is impossible. Computer aided calculation is a must. The most current computer programme for calculation is a spread-sheet type, namely Microsoft Excell. To maximize the spread-sheet capability : network data should be presented in a matrix form and this needs a matrix operation for calculation. Three forms of special matrix are presented in Table 3 below.

Table 3. Three Types of Special Matrix

Basic Matrix					Expanded Matrix						Indicative Matrix						
BM	1	2	3	4	EM	1	2	3	4	SR	IM	1	2	3	4	SR	IR
1					1						1						
2					2						2						
3					3						3						
4					4						4						
					SC						SC						
											IC						

Considering the type of data represented, the clarity, the practicality and the transportation network characteristics, the matrix has a following special convention regarding : its form and its cell.

- $n \times n$ matrix.
- diagonal cells : nodal data.
- non diagonal cells : link's data.
- 3 type of matrix : basic matrix, expanded matrix, indicative matrix.
- the matrix is equipped with title column, title row and a cell to put matrix name code.

Calculation method was developed around mathematical operation for constant, variabel, set and matrix. Example of three special matrix operations are presented below.

• Matrix Expansion :

$$\begin{aligned}
 me.M &= \chi m.M \\
 &= m.M + SR_i + SC_j + SM \\
 SR_i &= \sum m_{i,j} \\
 SC_j &= \sum m_{i,j} \\
 SM &= \sum m_{i,j} \text{ or } \sum SR_i \text{ or } \sum SC_j
 \end{aligned}$$

• Min-Plus Algebra Matrix Addition :

$$m.M = m.A \oplus m.B$$

$$m.M_{ij} = \min(a_{ij}, b_{ij})$$

- **Min-Plus Algebra Matrix Multiplication :**

$$m.M = m.A \otimes m.B$$

$$m.M_{ij} = \bigoplus_{p=1}^{p=n} (a_{ip} + b_{pj})$$

$$= \min_{p=1}^{p=n} (a_{ip} + b_{pj})$$

Afterward, four special calculation methods were developed. These four are noted specially, since these four are needed quite frequently. These four calculation methods are mentioned below.

- To indicate the existence of isolated nodes
- To indicate the existence of isolated network
- To indicate the existence of tree network
- Min-Plus Algebra Power Operation

Calculation of traffic flow itinerary expressed in terms of number trip and veh-km. Measuring in number of trip, the quality is expressed in percentage of number of trip well-served, which is number of trip well-served divided by number of total trip. Measuring in veh-km, the quality is expressed in percentage of veh-km well-served, which is value of veh-km well-served divided by total veh-km value.. Both are based on the ratio of shortest path length to direct distance. The calculation steps as presented below, practically all are done by special matrix operation

- Percentage of number of trip well served
 - Matrix of number of trip
 - Matrix of trip distribution
 - Matrix of trip production
 - Matrix of trip production factor
 - Matrix of trip attraction
 - Matrix of trip attraction factor
 - Matrix of impedance value
 - Matrix of direct distance
 - Matrix of node's x coordinate
 - Matrix of node's y coordinate
 - Impedance Function Coefficient
 - Matrix of trip well served
 - Matrix of ratio value between shortest distance to direct distance
 - Matrix of shortest path length
 - Matrix of direct distance
- Percentage of veh-km well served
 - Matrix of veh-km
 - Matrix of number of trip
 - Matrix of shortest path length
 - Matrix of veh-km well served
 - Matrix of ratio value between shortest distance to direct distance
 - Matrix of veh-km

4. Calculation Trial for Traffic Flow Itinerary Quality

4.1 The Trial Case

Bangkalan Regency road network was taken as a Test Case. Due to the road ability to be passed by different vehicle and due to the variety of vehicle circulate in the network, the road vehicle can be classified into six class : Truck Class 1, Truck Class 2, Truck Class 3, Multipurpose Vehicle, Sedan and Motorcycle. Traffic Flow Itinerary Quality has been calculated for all vehicle class. For the reason of limited text length, the paper cover only the Truck

Class 3 (T3). The map of Bangkalan Regency, the road network model and the network for 6 vehicle classes are presented in Fig. 2 to Fig. 4 below.



Fig. 2 : Bangkalan Regency.

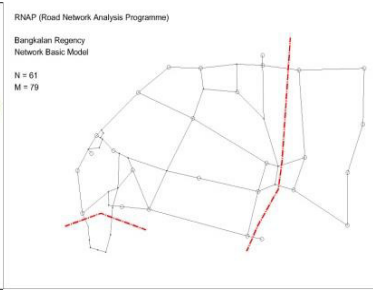


Fig. 3 : Basic Road Network Model.

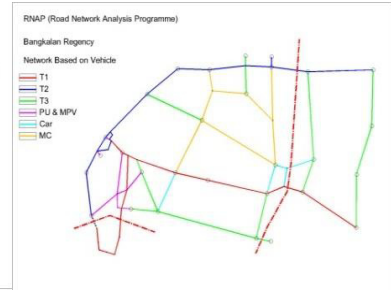


Fig. 4 : Network for Vehicle Type

4.2 Traffic Flow Itinerary Quality Calculation

Traffic Flow Itinerary Quality Calculation is presented for Truck Class 3. The quality is expressed in percentage of trip number which are well served and percentage of veh-km which are well served. A ratio of shortest path length to direct distance of 2 was taken as a value to distinguish between good itinerary and bad itinerary. The calculation follow the steps mentioned above. Example of two matrix is presented below.

Table 4. Matrix of Shortest Path for Truck Class 3

mSD22e_T3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22		
Kamal	1	0	41.4	38.3	42.8	52.4	44.0	40.9	32.4	27.4	7.0	15.5	19.9	22.8	34.3	37.0	37.4	43.6	37.8	0.0	13.7	1.8	39.0	708.5
John	2	41.4	0	6.1	17.4	47.0	58.4	38.9	18.9	14.0	34.4	28.4	11.5	38.6	49.2	68.4	68.3	34.5	48.7	0.0	30.8	44.0	88.8	888.3
Jeffrey	3	38.3	6.1	0	91.3	40.9	32.5	29.2	23.8	7.9	28.3	23.3	35.4	25.5	38.1	42.2	48.4	42.6	0.0	24.7	37.9	83.8	713.4	
Anthony	4	42.8	17.4	91.3	0	10.0	10.0	10.0	30.4	43.2	38.9	47.0	42.8	35.1	68.7	69.2	38.2	79.0	79.1	0.0	53.3	68.4	90.6	1,037.1
Henry	5	52.4	47.0	40.9	10.0	0	11.6	11.6	23.0	32.8	48.5	37.4	32.5	60.7	58.3	78.4	78.8	68.6	58.7	0.0	41.9	55.0	88.4	910.0
James	6	44.0	38.9	32.5	23.0	11.6	0	23.9	33.7	44.6	57.3	48.1	44.1	62.2	67.9	85.0	80.1	77.2	71.9	0.0	53.5	68.7	85.7	1,030.2
David	7	40.9	32.5	29.2	23.8	23.0	11.6	0	8.4	23.3	38.6	28.6	28.6	44.8	47.8	67.7	58.9	48.1	0.0	30.2	43.4	89.3	719.3	
Tyran Taweh	8	32.4	27.4	23.8	30.4	20.0	32.7	8.4	0	12.8	25.4	17.4	12.3	38.6	38.2	58.4	48.5	39.7	0.0	21.8	35.0	80.8	807.1	
Travis	9	27.4	14.0	7.9	43.2	32.9	44.6	23.3	12.9	0	20.4	12.4	7.4	28.6	31.2	54.3	54.3	40.5	34.7	0.0	18.8	30.2	55.8	587.8
Scott	10	10.0	34.4	28.3	38.9	48.5	37.1	23.3	28.4	20.4	0	8.6	13.9	13.9	27.4	30.5	30.4	38.7	38.8	0.0	8.8	9.8	52.0	615.2
Kevin	11	13.3	38.9	35.4	47.0	37.4	48.1	28.6	17.4	12.4	8.6	0	4.9	13.2	18.8	42.0	41.9	28.1	12.9	0.0	1.0	18.1	43.8	488.9
Jason	12	38.9	22.8	13.3	42.8	32.5	44.1	28.9	12.9	7.4	12.9	4.9	0	18.2	13.7	46.9	46.8	28.1	17.2	0.0	1.4	22.5	48.4	511.1
Archielex	13	37.0	37.4	34.5	43.6	48.1	62.2	67.9	85.0	77.2	71.9	13.2	18.2	0	0	1.6	38.7	38.6	14.9	0.0	0	18.3	33.4	392.2
Geiger	14	43.6	48.7	42.6	58.7	58.9	47.8	44.6	38.2	31.2	27.4	18.8	13.7	1.6	0	0	34.3	34.2	28.5	14.8	0	23.6	37.0	384.8
Shane	15	37.4	34.5	42.6	58.9	79.0	80.0	67.8	58.4	54.3	50.5	42.0	46.9	38.7	34.2	0	0	11.6	13.8	18.7	0	48.9	40.1	13.2
Therapuhm	16	37.0	48.1	42.2	68.7	78.8	80.1	67.7	58.3	54.3	50.4	46.8	38.6	34.2	11.6	0	0	13.7	19.8	0	48.9	40.2	1.6	877.9
Dejinh	17	43.6	48.1	42.6	78.8	68.6	77.2	67.8	48.5	40.5	36.7	28.1	23.3	14.9	20.5	13.8	13.7	0	0	0	31.1	48.3	25.3	766.6
Alampas	18	37.8	48.7	42.6	79.1	58.7	71.8	68.1	58.7	34.7	30.8	23.3	17.2	8.0	14.6	19.7	18.6	1.8	0	0	27.0	40.4	23.2	696.6
Shane	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Uy Bhangnan	20	13.7	30.8	24.7	52.3	41.9	53.5	30.2	23.8	16.8	8.0	5.0	8.4	18.2	13.8	46.9	46.9	28.1	17.3	0	0	18.4	48.4	547.9
PF Kamal	21	1.8	44.0	37.9	68.4	58.4	68.7	44.4	34.4	28.6	20.0	1.0	48.1	13.3	11.4	0	0	10.0	48.9	0	0	61.6	709.3	
TL Degeyrie	22	39.0	88.8	83.8	90.8	80.4	82.7	89.3	80.8	58.8	52.0	49.5	46.8	38.2	38.8	13.2	1.6	13.3	21.2	0	0	48.4	81.8	0
Sum	23	708.5	888.3	713.4	1,037.1	910.0	1,030.2	719.3	807.1	877.8	615.2	488.9	511.1	589.2	489.2	489.2	489.2	489.2	489.2	0	547.8	759.3	1,000.0	1,000.0

Table 5. Matrix of Itinerary Not Well Served

mSD22e_T3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The calculation results is then mapped into these three graphical mdel : the road network, the total desired lines and the desired lines not well served. These models are presented in Fig. 5 – Fig. 7 below.

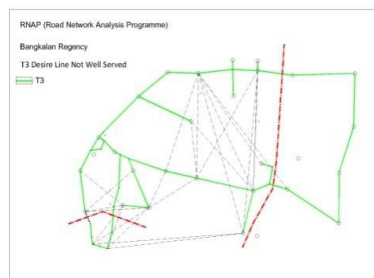


Fig. 5. T3 Road Network.

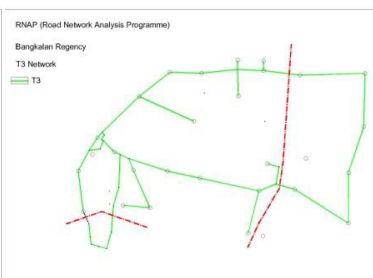


Fig. 6 T3 Desired Lines.

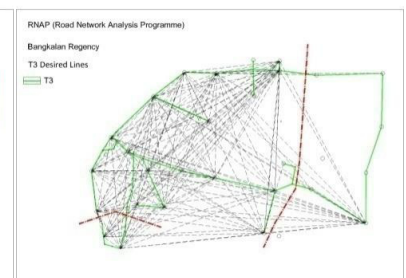


Fig. 7 T3 Desired Lines Not Well Served..

The whole calculation result is presented in Tabel 4 below. It can be noted that the itinerary quality measured in number of trips well served seems good, 91%. But this quality measured in veh-km is worse, 83%. The two measure are therefore very important.

Tabel 6 Calculation Result of Truck Class 3 Traffic Flow Itinerary Quality

	Total	Well Served	Difference	Ratio
Number of Trip	537	490	47	0.91
Veh-Km	14,897.29	12,368.34	2,528.96	0.83

5. Conclusions

The objective is attained, the traffic flow itinerary has been formulated together with its calculation method. Bangkalan Regency T3 Network has been taken as a Test Case. Traffic Flow Itinerary Quality is measured in percentage number of trip well served and percentage of vehicle-km well served. Both two are important.

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